

C) PARABOLA

The parabola in the Level 3 Conics course is similar in structure to the parabola studied in Level 1 and Level 2 Maths, but this time it is on its side.

The basic equations are:

$$y^2 = 4ax \quad \text{vertex at } (0, 0)$$

$$(y - k)^2 = 4a(x - h) \quad \text{vertex at } (h, k)$$

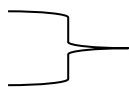
And **a** is the **focus**, and the distance of the focus from the vertex is called the **focal length**.

To do a quick sketch when given the equation of the parabola, mark 3 points:

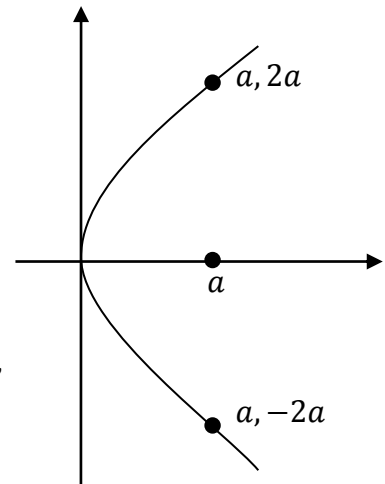
Point 1: vertex

Point 2: $(a, 2a)$

Point 3: $(a, -2a)$



Taking into account translation



COMPLETING THE SQUARE FOR A PARABOLA

Examples:

1) Sketch $y^2 - 4y - 4x - 8 = 0$.

$$y^2 - 4y - 4x - 8 = 0$$

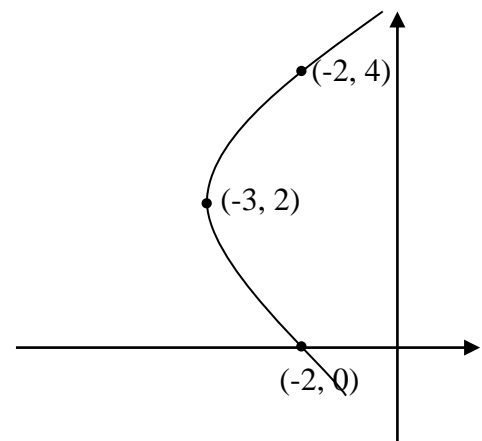
$$y^2 - 4y = 4x + 8$$

$$\left[y^2 - 4y + \left(\frac{-4}{2}\right)^2 \right] = 4x + 8 + \left(\frac{-4}{2}\right)^2$$

$$[y^2 - 4y + 4] = 4x + 8 + 4$$

$$(y - 2)^2 = 4x + 12$$

$$(y - 2)^2 = 4(x + 3)$$



Therefore, vertex = $(-3, 2)$. Coefficient of $x = 4a$, i.e. $4 = 4a$, therefore $a = 1$.

Quick sketch: Point 1 = vertex $(-3, 2)$, Point 2 = $(-2, 4)$, Point 3 = $(-2, 0)$.

2) Sketch $y^2 - 6x - 4y - 14 = 0$.

$$y^2 - 6x - 4y - 14 = 0$$

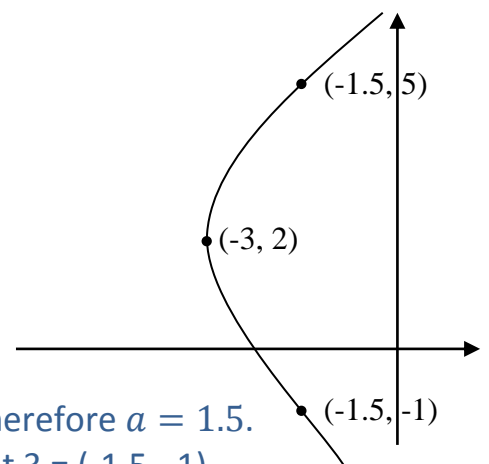
$$y^2 - 4y = 6x + 14$$

$$\left[y^2 - 4y + \left(\frac{-4}{2}\right)^2 \right] = 6x + 14 + \left(\frac{-4}{2}\right)^2$$

$$[y^2 - 4y + 4] = 6x + 14 + 4$$

$$(y - 2)^2 = 6x + 18$$

$$(y - 2)^2 = 6(x + 3)$$



Therefore vertex = $(-3, 2)$. Coefficient of $x = 4a$, i.e. $6 = 4a$, therefore $a = 1.5$.

Quick sketch: Point 1 = vertex $(-3, 2)$, Point 2 = $(-1.5, 5)$, Point 3 = $(-1.5, -1)$.

How to form an equation, given the graph of a parabola:

- 1) Start with the basic equation of a parabola: either $y^2 = 4ax$ or $(y - k)^2 = 4a(x - h)$.
- 2) Pick a point on your parabola that is NOT the vertex.
- 3) Substitute the coordinates of that point into the equation for a parabola, and solve for a .
- 4) Write your final equation.

Worksheet, Delta Ex 37.5 pg 371 Q1 – 4, Extension: Q5 – 8